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# On the interactions of slender ships in shallow water

By RONALD W. YEUNG
Department of Ocean Engineering, Massachusetts Institute of Technology, Cambridge

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The unsteady hydrodynamic interaction of two bodies moving in a shallow fluid is examined by applying slender-body theory. The bodies are assumed to be in each other's far field and the free surface is assumed to be rigid. By matched asymptotics, the inner and outer problems are formulated and a pair of coupled integro-differential equations for determining the unknown cross-flows is derived. The degree of coupling is shown to be related to a bottom-clearance parameter. Expressions are given for the unsteady sinkage force, trimming moment, sway force, and yaw moment. Numerical calculations for two weakly coupled cases are presented. One corresponds to the interaction of a stationary body with a passing one, the other to the interaction of two bodies moving in a steady configuration. Theoretical results are compared with existing experimental data.

## 1. Introduction

The subject of hydrodynamic interaction between bodies moving in close proximity has been of classical interest, for there are practical situations in which interaction forces and moments play a dominant role. Proximity manoeuvres of naval vessels, collision-course encounters of ships, and congested vessel traffic in harbours are a few of such situations. The interaction phenomenon is generally aggravated by the effects of shallow water. The advent of super-tankers has made the consideration of these effects imperative.

A brief review of past analytical work on hydrodynamic interactions between ships will be given. The problem of two spheroids in tandem motion in a deep fluid was examined by Havelock as early as 1949. Exploiting the slender-body assumptions, Newman (1965) presented closed-form results for a spheroid moving near a wall. Wang (1975) developed a slender-body model for the prediction of mooring forces on a stationary ship due to a passing one. The effects of finite depth were also examined under the assumption that the depth was of the same order as the ship length. Tuck & Newman (1974) considered a similar approach, but allowed both ships to have non-zero speeds. Collatz (1963) solved the exact potential-flow problem of two elliptical cylinders in unsteady motion. King (1977) considered the same problem but with the effects of circulation included. In these last two studies, the assumption of two-dimensionality is equivalent to representing the vessels by airfoils. A related mathematical model used by Dand (1976) appears to give excessively large forces and moments when compared with experimental values. All studies cited are based on inviscid-flow theory with a rigid free-surface condition.

The approach used in this paper is based on the theory of matched asymptotics. The three-dimensional problem of two ships moving in a shallow fluid is first recast into

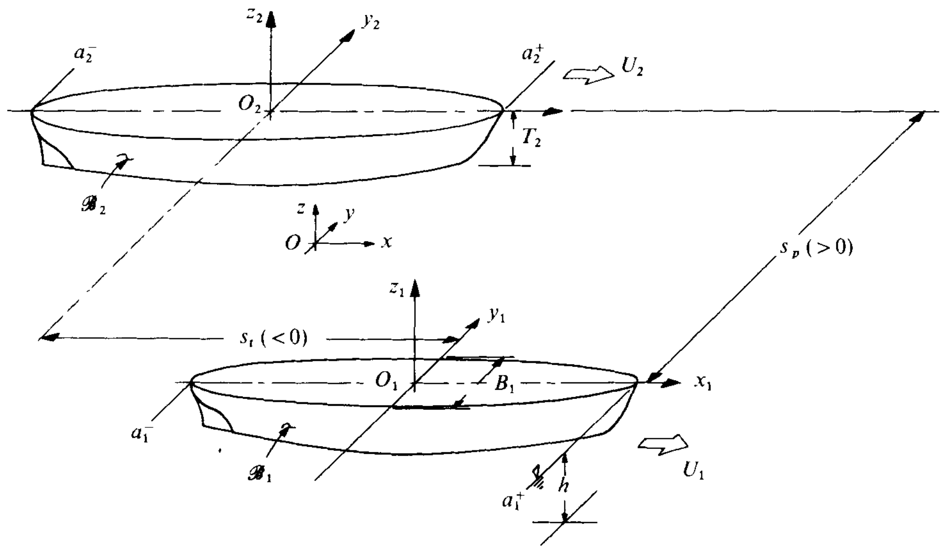


FIGURE 1. Co-ordinate systems.

two inner problems and one outer problem. Section 2 describes all essential conditions associated with the outer problem. Section 3 considers the inner problem and its outer limit. The matching process is carried out in §4. The interaction hydrodynamic forces and moments are next derived for both the vertical and the horizontal plane. The solutions for two special cases are considered in §6. Some numerical results are presented and discussed in §7.

## 2. Problem formulation

Consider two vessels designated as bodies 1 and 2 moving at speeds $U_1$ and $U_2$ in an inviscid fluid of depth $h$. At the outset, the free surface is assumed to be rigid, which implies that the effects of waves are neglected. This is known to be a plausible assumption if the depth Froude number is small, i.e. $U_j/(gh)^{\frac{1}{2}} = o(\epsilon)$ $(j = 1, 2)$, where $g$ is the acceleration due to gravity and $\epsilon$ a small parameter. Therefore the rigid free-surface problem formulated below may be regarded as the infinite-gravity limit of the more general problem where wave effects are important, or as the leading-order problem corresponding to a low-speed perturbation analysis. Such a free-surface condition reduces the problem to the determination of the flow about the two bodies and their images above the free surface, sandwiched between parallel walls a distance $2h$ apart.

Let $O_j x_j y_j z_j$ $(j = 1, 2)$ be two moving co-ordinate systems attached to the vessels as shown in figure 1. Let $Oxyz$ be a third co-ordinate system fixed in space. If $\nabla\phi(x, y, z, t)$ denotes the absolute velocity of the fluid particles due to the motion of the bodies, then the following 'exact' boundary-value problem for the velocity potential $\phi$ can be formulated:

$$\nabla^2\phi(x, y, z, t) = 0, \tag{1}$$

$$[\partial\phi/\partial n]_{\mathscr{B}_1(t)} = U_1(n_x)_1, \quad [\partial\phi/\partial n]_{\mathscr{B}_2(t)} = U_2(n_x)_2, \tag{2}$$

$$[\partial\phi/\partial z]_{z=\pm h} = 0, \tag{3}$$

where $(n_x)_j$ represents the $x$ component of the interior normal to the body surface $\mathscr{B}_j(t)$. The relation between the two moving co-ordinate systems is straightforward, viz.

$$x_1 = x_2 + s_t(t), \quad y_1 = y_2 + s_p, \tag{4}$$

where $s_t$ is the stagger and $s_p$ the separation, both measured with respect to the origin $O_1$. The following assumptions of slenderness are now introduced to simplify the problem:

$$L_j = O(1), \quad B_j = O(\epsilon), \quad T_j = O(\epsilon) \quad (j = 1, 2), \tag{5}$$

$$h = O(\epsilon), \quad s_p = O(1). \tag{6), (7}$$

Equations (5) and (6) justify the application of slender-body theory. Assumption (7) permits the use of the so-called outer representation of one ship when the observer is near the other. The shallow-fluid assumption (6) also implies that, if the present zero-Froude-number theory were to be applied to situations in which $F_h = o(\epsilon)$, the conventional Froude number based on ship length could be $o(\epsilon^{\frac{3}{2}})$.

The boundary-value problem (1)–(3) will now be recast into two inner problems, one for each body, and an outer problem. This procedure is similar to that used by Tuck (1966). Hence only a brief summary will be given.

Let $Y_j$ and $Z_j$ be the inner variables near the $j$th body stretched according to $Y_j = y_j/\epsilon$ and $Z_j = z_j/\epsilon$. Then, to leading order, the following problems for the inner potential $\Phi_j$ $(j = 1, 2)$ can be easily derived by using (1)–(3):

$$(\partial^2/\partial Y_j^2 + \partial^2/\partial Z_j^2)\, \Phi_j(Y_j, Z_j, x_j, t) = 0 \quad \text{for} \quad (Y_j, Z_j) \quad \text{near} \quad \mathscr{B}_j, \tag{8}$$

$$[\partial \Phi_j/\partial N]_{\Sigma_j^B} = U_j n_x(Y_j, Z_j) \quad (j = 1, 2), \tag{9}$$

$$[\partial \Phi_j/\partial Z_j]_{z_j = \pm h} = 0. \tag{10}$$

Here $\mathbf{N}$ represents the unit two-dimensional interior normal to the section contour $\Sigma_j^B$. It can be seen that the inner potentials $\Phi_j$ satisfy Laplace's equation in the cross-plane, a flux condition on the cross-section contour and a no-flux condition on the walls. The time dependence of $\Phi_j$ arises from the fact that the flow incident upon a particular section of the $j$th ship is a function of the stagger $s_t$, which changes with time. This inner problem is illustrated in figure 2, wherein it is noteworthy that the cross-flow $V^*(x_j, t)$ is unknown and its order of magnitude will depend on the blockage characteristics of the cross-section. This point will be addressed in a later section.

For the outer problem, $x$ and $y$ are both $O(1)$, however $z$ is $O(\epsilon)$. If $\phi$ is written in terms of an expansion of the form

$$\phi = \phi^{(1)}(x, y, z, t) + \phi^{(2)} + \phi^{(3)} + \phi^{(4)} + \ldots,$$

where it is assumed that $\phi^{(n+1)} = o(\phi^{(n)})$ for all $n$, then (1) yields

$$\phi_{zz}^{(1)} = 0, \quad \phi_{zz}^{(2)} = 0, \quad \phi_{zz}^{(3),(4)} = -\nabla_{x,y}^2 \phi^{(1),(2)}.$$

$\phi^{(1)}$ and $\phi^{(2)}$, however, cannot depend on $z$ because of (3). If (3) is next applied to $\phi^{(3)}$ and $\phi^{(4)}$, the governing equations for $\phi^{(1)}$ and $\phi^{(2)}$ can be obtained:

$$\nabla_{x,y}^2 \phi^{(i)}(x, y, t) = 0 \quad (i = 1, 2). \tag{11}$$

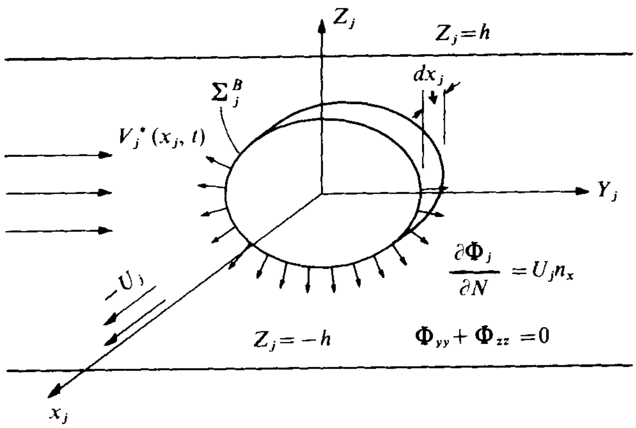


FIGURE 2. The flow near the hull in a cross-flow plane.

Therefore the first two terms of the outer expansion satisfy Laplace's equation in the horizontal plane. The body boundary condition (2) is not applicable to the outer problem since to an observer $O(L)$ away from the bodies they would appear to have collapsed onto a line.

Only the solution of the lowest-order problem will be sought in this paper. With that as the understanding, the superscript 1 will henceforth be omitted. The two-dimensionality of the outer flow suggests the following representation of $\phi$ in terms of line-source and line-vortex distributions:

$$\phi(x,y,t) = \sum_{j=1}^{2}\left\{\frac{1}{2\pi}\int_{a_j^-(t)}^{a_j^+(t)} m_j(\xi,t)\log\left[(x-\xi)^2+\left(y-(-1)^j\frac{s_p}{2}\right)^2\right]^{\frac{1}{2}} d\xi \right.$$
$$\left. +\frac{1}{2\pi}\int_{-\infty}^{a_j^+(t)} \gamma_j(\xi,t)\tan^{-1}\left[\frac{y-\frac{1}{2}(-)^j s_p}{x-\xi}\right] d\xi\right\}, \quad (12)$$

where the longitudinal axes of the two ships are assumed to be at $y = \pm\frac{1}{2}s_p$ and $[a_j^-, a_j^+]$ denotes the instantaneous location of the $j$th ship. In (12), the branch cut of the arctangent function should be chosen downstream of the translating vortices. The unknown source and vortex strengths $m$ and $\gamma$ cannot be determined from the outer problem alone. However, by *matching* the inner and outer solutions properly the necessary relations can be obtained.

It is worthwhile to note that the unsteadiness of the problem gives rise to a vortex distribution in the wake of the bodies. Inasmuch as the outer flow is two-dimensional, existing analysis and conditions concerning the unsteady motion of a two-dimensional airfoil (see Garrick 1957) are applicable here. In particular, the linearized pressure-continuity condition across the two-dimensional wake is given by

$$p^+-p^- = \partial[\phi(x,y_j=0^+,t)-\phi(x,y_j=0^-,t)]/\partial t = 0 \quad \text{for} \quad x < a_j^-(t). \quad (13)$$

On the other hand, by definition,

$$\gamma_j(x,t) = \partial\phi(x,y_j=0^+,t)/\partial x - \partial\phi(x,y_j=0^-,t)/\partial x. \quad (14)$$

Equation (13) thus implies

$$\partial\gamma_j(x,t)/\partial t = 0 \quad \text{or} \quad \gamma_j(x,t)=\gamma_j(x) \quad \text{for} \quad x < a_j^-(t). \quad (15)$$

Whence any vorticity shed in the wake remains constant in time and depends only on the wake co-ordinate in the inertial frame of reference. The rate at which vorticity is being shed by each body can be derived from Kelvin's theorem, which can be interpreted as follows. The circulation due to any material contour that encloses each body individually should be zero at any time since the initial circulation when the bodies were far apart was identically zero. Thus any gain in the bound circulation $\Gamma_j$ of the body must be compensated by a shedding of vorticity of opposite sign. Whence, according to von Kármán & Sears (1938), it is possible to show that

$$d\Gamma_j/dt = -U_j\gamma_j(a_j^-(t)). \tag{16}$$

Finally, to ensure uniqueness of the solution, we note the important subsidiary trailing-edge condition, which requires that the flow at the trailing edge be smooth. This Kutta condition can be stated as

$$\lim_{x\to a_j^- -0}\gamma_j(x,t) = \lim_{x\to a_j^- +0}\gamma_j(x,t) \quad \text{for all } t. \tag{17}$$

Numerical procedures for solving two-dimensional outer problems involving multiple bodies have been presented by Giesing (1968) and recently by King (1977).

## 3. The inner problems

It is convenient to decompose the inner problem defined by (8)–(10) into two component potentials, one associated with the forward motion and the other with the lateral flow, as follows:

$$\Phi_j(Y_j, Z_j, x_j, t) = \Phi_j^{(1)} + V_j^*(x_j, t)\,\Phi_j^{(2)} + f_j(x_j, t) \tag{18}$$

with

$$\partial\Phi_j^{(1)}/\partial N = U_j n_x(Y_j, Z_j) \quad \text{on} \quad \Sigma_j^B, \tag{19}$$

$$\partial\Phi_j^{(2)}/\partial N = 0 \quad \text{on} \quad \Sigma_j^B. \tag{20}$$

In addition, both potentials should satisfy (10). It is clear that the inhomogeneous boundary condition is satisfied by $\Phi_j^{(1)}$ and that $\Phi_j^{(2)}$ corresponds to the problem of unit lateral flow about a cylinder located between walls. The non-uniqueness of the inner problem asserts itself in the form of the unknown functions $V_j^*$ and $f_j(x_j, t)$.

By examining (9), one notes that $\Phi_j^{(1)}$ is $O(\epsilon^2)$ with respect to outer variables. The magnitude of $\Phi_j^{(2)}$ can be estimated from the behaviour of the added mass of the cylinder. The following asymptotic formula for a rectangular cross-section with a small bottom clearance was given by Flagg & Newman (1971):

$$\begin{aligned} A_j(x) &= 4h^2\left[\frac{B_j(x)}{2\delta_j h} + \frac{2}{\pi}(1-\log 4\delta_j) - \frac{B_j}{h}(1-\delta_j) + O(\delta_j^2)\right] \\ &= O(\epsilon^2/\delta_j), \end{aligned} \tag{21}$$

where $A_j(x)$ is the 'double-body' added-mass coefficient and $\delta_j = (h - T_j)/h$. Hence $\Phi_j^{(2)}$ is $O(V^*\epsilon\delta_1^{-1})$. In arriving at the second equality in (21), it was necessary to invoke (5) and (6). As far as completing the matching process is concerned, it is not necessary to solve for these potentials in detail, but a knowledge of their outer behaviour is

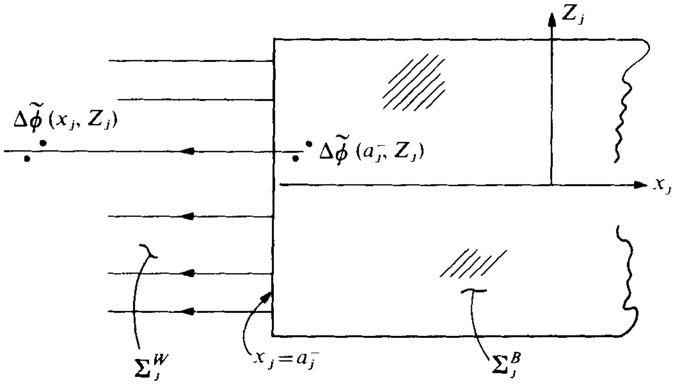


FIGURE 3. Kutta condition at an abrupt trailing edge.

required. In some intermediate region where $Y_j = O(L_j)$, (18) can easily be shown to be

$$\lim_{Y_j \to \pm L_j} \Phi_j(Y_j, Z_j, x_j, t) = \underset{O(\epsilon)}{(-U_j S_j'(x_j)/4h)\,|Y_j|} + \underset{O(V_j^* \epsilon/\delta_j)}{V_j^*[Y_j \pm C_j(x_j)]} + \underset{O(f_j)}{f_j(x_j, t)} \quad \text{for} \quad a_j^- \leqslant x_j \leqslant a_j^+ \tag{22}$$

with the order of $f_j$ to be determined from matching. In (22), $S_j$ is the area enclosed by $\Sigma_j^B$ and $C_j$ is the blockage constant used frequently to characterize a lateral flow about cascades (Sedov 1965):

$$C_j(x) = (A_j(x) + S_j(x))/4h. \tag{23}$$

Next, consider $x_j$ lying in the wake region of the $j$th body. Clearly, the term $\Phi_j^{(1)}$ in (18) should be omitted. If the body is assumed to terminate in the form of a thin and abrupt trailing edge (figure 3) a vortex sheet will be shed downstream of this edge. By Green's theorem, the perturbation potential $\tilde{\phi}$ associated with the lateral flow potential $\Phi_j^{(2)}$ ($j = 1, 2$) can be written as

$$2\pi\tilde{\phi}(Y, Z; x) = \int_{\Sigma_j^B} [\tilde{\phi}(0^-, Z) - \tilde{\phi}(0^+, Z)] \frac{\partial G}{\partial Y} dZ + \oint_{\Sigma_j^B} \left(\frac{\partial Y}{\partial N}\right) G\, ds \quad \text{for} \quad x = a_j^- + 0, \tag{24}$$

$$2\pi\tilde{\phi}(Y, Z; x) = \int_{\Sigma_j^W} [\tilde{\phi}(0^-, Z) - \tilde{\phi}(0^+, Z)] \frac{\partial G}{\partial Y} dZ - \oint_{\Sigma_j^W} \frac{\partial \tilde{\phi}}{\partial N} G\, ds \quad \text{for} \quad x = a_j^- - 0. \tag{25}$$

In (25), $ds$ is an infinitesimal arc-length element and $G$ is a Green function satisfying the wall conditions. In the spirit of slender-body theory, Newman & Wu (1972) have proposed a Kutta condition of a weak type to be imposed at such a trailing edge. This condition is congruent to requiring the potential be locally continuous at the juncture of $\Sigma_j^W$ and $\Sigma_j^B$. Thus it follows that the first terms on the right-hand sides of (24) and (25) are identical. From geometrical considerations, the second term of both equations vanishes. Hence one arrives at the following outer behaviour of the inner potential in the wake:

$$\lim_{Y_j \to \pm L_j} \Phi_j^{(2)} = V_j^*(a_j^-, t)\,[Y_j \pm A_j(a_j^-)] \quad \text{for all} \quad x_j < a_j^-, \tag{26}$$

where $\hat{t}$ is the retarded time defined by $\hat{t} \equiv t + (x_j - a_j^-)/U_j$. The retarded time arises as a result of the linearized dynamic boundary condition on the vortex sheet:

$$\partial'\Phi_j/\partial t - U_j \,\partial\Phi_j/\partial x_j = 0, \tag{27}$$

where $\partial'/\partial t$ indicates differentiation with respect to time in the moving frames. It is of interest to note that the presence of the vortex sheet offers an apparent added-mass effect even though there is no physical obstruction in the fluid.

## 4. Matching

In order to match the outer solution with the inner solution described by (22) and (26), it will be necessary to obtain an inner expansion of (12) near each body. For clarity of exposition, consider first the case when $y_1$ $(= y - \frac{1}{2}s_p)$ is small. In the coordinate system of body 1,

$$\begin{aligned}\phi(x_1, y_1, t) &= \phi(x_1, 0^\pm, t) + \partial\phi(x_1, 0^\pm, t)/\partial y\, y_1 + O(y_1^2)\\ &= \phi(x_1, 0^\pm, t) + V_{21}(x_1, t)\, y_1 \pm \frac{m_1(x_1, t)}{2}|y_1| + \frac{1}{2\pi}\int_{-\infty}^{a_1^+} \frac{\gamma_1(\xi_1, t)}{x_1 - \xi_1}\, d\xi_1\, y_1 + O(y_1^2)\\ &\qquad \text{for} \quad a_1^- \leqslant x \leqslant a_1^+, \end{aligned} \tag{28}$$

where $V_{21}$ is the normal velocity induced by body 2 on the axis of body 1,

$$V_{21}(x_1, t) = \frac{-1}{2\pi}\int_{a_2^-}^{a_2^+} m_2(\xi_2, t)\, \frac{s_p}{(x_2 - \xi_2)^2 + s_p^2}\, d\xi_2 + \frac{1}{2\pi}\int_{-\infty}^{a_2^+} \gamma_2(\xi_2, t)\, \frac{x_2 - \xi_2}{(x_2 - \xi_2)^2 + s_p^2}\, d\xi_2, \tag{29}$$

and

$$\begin{aligned}\phi(x_1, 0^\pm, t) &= \frac{1}{2\pi}\int_{a_1^-}^{a_1^+} m_1(\xi_1, t) \log|x_1 - \xi_1|\, d\xi + \frac{1}{2\pi}\int_{a_2^-}^{a_2^+} m_2(\xi_2, t) \log[(x_2 - \xi_2)^2 + s_p^2]\, d\xi_2\\ &\quad + \int_{-\infty}^{a_2^+} \gamma_2(\xi_2, t) \tan^{-1}\left(\frac{-s_p}{x_2 - \xi_2}\right) d\xi_2 \pm \frac{1}{2}\int_{x}^{a_1^-} \gamma_1(\xi_1, t)\, d\xi_1. \end{aligned} \tag{30}$$

The term $m_1(x_1, t)$ in (28) is inderstood to be zero when $x_1 < a_1^-$.

A straightforward comparison of terms of (28) with those of a similar nature in (22) and (26) yields the following four relations:

$$m_1(x_1, t) = -U_1 S_1'(x_1)/2h, \tag{31}$$

$$V_1^*(x_1, t) = V_{21}(x_1, t) + \frac{1}{2\pi}\int_{-\infty}^{a_1^+} \frac{\gamma_1(\xi_1, t)}{x_1 - \xi_1}\, d\xi_1, \tag{32}$$

$$V_1^* C_1 = \frac{1}{2}\int_{x_1}^{a_1^+} \gamma_1(\xi_1, t)\, d\xi_1, \tag{33}$$

$$\begin{aligned} f_1(x_1, t) &= \frac{-U_2}{4\pi h}\int_{a_1^-}^{a_1^+} S_1'(\xi_1) \log|x_1 - \xi_1|\, d\xi_1 + \frac{1}{2\pi}\int_{a_2^-}^{a_2^+} m_2(\xi_2, t) \log[(x_2 - \xi_2)^2 + s_p^2]\, d\xi_2\\ &\quad + \frac{1}{2\pi}\int_{-\infty}^{a_2^+} \gamma_2(\xi_2, t) \tan^{-1}\left(\frac{-s_p}{x_2 - \xi_2}\right) d\xi_2. \end{aligned} \tag{34}$$

Note that differentiation of (33) with respect to $x_1$ yields

$$\gamma_1 = -2\partial[V_1^*(x_1, t)\, C_1(x_1)]/\partial x_1 \quad \text{for} \quad a_1^- \leqslant x_1 \leqslant a_1^+, \tag{35}$$

$$\gamma_1 = -2A_1(a_1^-)\, \partial V_1^*(a_1^-, \hat{t})/\partial x_1 \quad \text{for} \quad x_1 < a_1^-. \tag{36}$$

A completely analogous procedure can also be carried out for body 2. The resulting equations are similar and will not be repeated here. In both cases, it is noteworthy that the source strength matches as in the case of a single body considered by Tuck (1966) whereas the vortex strength cannot be determined until (32) is solved. The physical interpretation of (32) is as follows: the cross-flow that body 1 sees is that generated by the adjacent body plus the normal velocity induced by its own vortex distribution. The axial flow $f_1'(x,t)$ along the body, which will determine its sinkage and trim, is generated not only by the source distribution of the body itself, but also by the singularity distribution of the adjacent body.

The following coupled integro-differential equations for $V_1^*$ and $V_2^*$ can now be derived by making use of (35), (36) and the analogous equations for body 2:

$$V_1^*(x_1,t)+\frac{1}{\pi}\int_{-\infty}^{a_1^+}\frac{(V_1^*C_1)'}{x_1-\xi_1}d\xi_1+\frac{1}{\pi}\int_{-\infty}^{a_2^+}\frac{(V_2^*C_2)'(x_2-\xi_2)\,d\xi_2}{(x_2-\xi_2)^2+s_p^2}=\frac{U_2}{4\pi h}\int_{a_2^-}^{a_2^+}\frac{S_2'(\xi_2)s_p\,d\xi_2}{(x_2-\xi_2)^2+s_p^2},$$

$$O(V_1^*)\qquad O(V_1^*\epsilon\delta_1^{-1})\qquad O(V_2^*\epsilon\delta_2^{-1})\qquad O(\epsilon)\qquad (37)$$

$$V_2^*(x_2,t)+\frac{1}{\pi}\int_{-\infty}^{a_2^+}\frac{(V_2^*C_2)'}{x_2-\xi_2}d\xi_2+\frac{1}{\pi}\int_{-\infty}^{a_1^+}\frac{(V_1^*C_1)'(x_1-\xi_1)\,d\xi_1}{(x_1-\xi_1)^2+s_p^2}=\frac{-U_1}{4\pi h}\int_{a_1^-}^{a_1^+}\frac{S_1'(\xi_1)\,s_p\,d\xi_1}{(x_1-\xi_1)^2+s_p^2},$$

$$O(V_2^*)\qquad O(V_2^*\epsilon\delta_2^{-1})\qquad O(V_2^*\epsilon\delta_1^{-1})\qquad O(\epsilon)\qquad (38)$$

where the primes denote differentiation with respect to the space variable. The order of each term is given below the equations.

One observes that (37) and (38) resemble a pair of coupled Prandtl lifting-line equations with $V^*C$ playing the role of bound vorticity on a large-aspect-ratio wing. Note further that, although these equations have to be solved for each value of $t$, it is necessary to determine the value of $V^*$ in only $[a_1^-, a_1^+]$ and $[a_2^-, a_2^+]$, since the quantity $(V^*C)'$ in the wake is related to $V^*$ at the trailing edge by (36).

For two bodies of the same slenderness ratio, the degree to which they interact depends on the magnitude of $C_1$ and $C_2$. Examination of (37) and (38) shows that the possible situations may be divided into the following three cases.

*Case* 1: $\delta_1 = O(1), \quad \delta_2 = O(1).$

This corresponds to the physical situation where the bottom clearances of both bodies are the same *order* as their drafts. Thus to the leading-order approximation, one notices from (37) and (38) that the cross-flow according to a consistent analysis is simply that generated by the source distribution of the adjacent body and that such a cross-flow is $O(\epsilon)$.

*Case* 2: $\delta_1 = O(\epsilon), \quad \delta_2 = O(1).$

The first equation is weakly coupled to the second. The third term of (37) can be omitted when solving for the flow about body 1. $V_1^*$, which must be obtained by solving (37), is seen to be $O(\epsilon)$. On the other hand $V_2^*$, which is also $O(\epsilon)$, can be determined in closed form from the right-hand side of (38) and the solution for $V_1^*$. These arguments, of course, apply also to the converse case $\delta_1 = O(1)$, $\delta_2 = O(\epsilon)$.

*Case* 3: $\delta_1 = O(\epsilon), \quad \delta_2 = O(\epsilon).$

No simplifications are possible for this case; the coupled equations have to be solved simultaneously. An alternative form of (37) and (38) which may be more amenable to numerical solution can be obtained by using the vorticity strengths $\gamma_j$ as the unknown functions instead of the $V_j^*$:

$$\frac{1}{C_1(x_1)}\int_{x_1}^{a_1^+}\gamma_1(\xi_1,t)\,d\xi_1-\frac{1}{\pi}\int_{-\infty}^{a_1^+}\frac{\gamma_1(\xi_1)}{x_1-\xi_1}d\xi_1-\frac{1}{\pi}\int_{-\infty}^{a_2^+}\frac{\gamma_2(\xi_2)\,(x_2-\xi_2)}{(x_2-\xi_2)^2+s_p^2}d\xi_2 = \frac{U_2}{2\pi h}\int_{a_2^-}^{a_2^+}\frac{S_2'(\xi_2)\,s_p}{(x_2-\xi_2)^2+s_p^2}d\xi_2, \quad (39)$$

$$\frac{1}{C_2(x_2)}\int_{x_2}^{a_2^+}\gamma_2(\xi_2,t)\,d\xi_2-\frac{1}{\pi}\int_{-\infty}^{a_2^+}\frac{\gamma_2(\xi_2)}{x_2-\xi_2}d\xi_2-\frac{1}{\pi}\int_{-\infty}^{a_1^+}\frac{\gamma_1(\xi_1)\,(x_1-\xi_1)}{(x_1-\xi_1)^2+s_p^2}d\xi_1 = \frac{-U_1}{2\pi h}\int_{a_1^-}^{a_1^+}\frac{S_1'(\xi_1)\,s_p}{(x_1-\xi_1)^2+s_p^2}d\xi_1, \quad (40)$$

which are two coupled Volterra equations of the first kind.

Finally, it seems worthwhile to show that the inner and outer Kutta conditions described in §§2 and 3 are indeed consistent. From (16), (17) and (35),

$$\gamma_j(a_j^-+0,t)=\frac{-2}{U_j}\frac{d}{dt}\int_{a_j^-}^{a_j^+}(V_j^*C_j)'\,dx_j=-2\frac{C_j(a_j^-)}{U_j}\frac{\partial'}{\partial t}V_j^*(a_j^-,t)\quad\text{for}\quad C_j(a_j^+)=0, \quad (41)$$

while from the inner-field condition (27) and the matching condition (36)

$$\gamma_j(x_j,t)=-2C_j(a_j^-)\frac{\partial}{\partial x_j}V_j^*(a^-,\hat{t}(x_j))=\frac{-2C_j(a_j^-)}{U_j}\frac{\partial'}{\partial t}V_j^*(a_j^-,t)\Big|_{t=\hat{t}}\quad\text{for}\quad x_j<a_j^-, \quad (42)$$

which is clearly identical to (41) in the limit $x_j = a_j^- - 0$. Note further that, if the flow is steady or if the trailing edge is not fin-like $\gamma_j$ vanishes at this edge.

## 5. The interaction hydrodynamic force and moment

Of primary interest in the physical problem being studied are the lateral force and moment on each body. There exists also a sinkage force and a trimming moment acting on the 'wetted' half of the double body. The desired hydrodynamic force or moment can be conveniently obtained from the inner field once $V^*$ is known from solving (37) and (38). If one makes use of the inner potential (18) and the unsteady Bernoulli equation, the following expression for the fluid pressure $p(Y,Z;x,t)$ acting on, say, body 1 can be derived:

$$p(Y_1,Z_1;x_1,t)=\epsilon p_1(x_1,t)+\epsilon^2p_2(x_1,t)+\epsilon^2P(Y_1,Z_1;x_1,t)+O(\epsilon^3), \quad (43)$$

where

$$p_1(x_1,t)/\rho=-\partial'f_1(x_1,t)/\partial t+U_1f_1', \quad (44)$$

$$p_2(x_1,t)/\rho=-\tfrac{1}{2}(U_1f_1')^2, \quad (45)$$

$$P_2(Y_1,Z_1;x_1,t)/\rho=-\frac{\partial' V_1^*}{\partial t}\Phi_1^{(2)}+U_1\frac{\partial}{\partial x}\langle\rangle-\frac{1}{2}\left[\left(\frac{\partial}{\partial Y_1}\langle\rangle\right)^2+\left(\frac{\partial}{\partial Z_1}\langle\rangle\right)^2\right], \quad (46)$$

the symbol $\langle\rangle$ denoting the quantity $\langle\Phi_1^{(1)}+V_1^*\Phi_1^{(2)}\rangle$. The time differentiation is understood to be with respect to the moving co-ordinate system. In these expressions, it is worthwhile to keep in mind that $V_1^* = O(\epsilon)$ and $\Phi_1^{(2)}$ could be of an order smaller than $\epsilon$ if the bottom clearance were not small. Note that, because of (34), (31) and (35), $f_1(x_1, t)$ is $O(\epsilon)$. To the leading order, it is evident that the sinkage force $-\mathscr{Z}$ and the trimming moment $\mathscr{M}$ (about $O_1$) are due to $p_1$:

$$\begin{pmatrix}\mathscr{Z}_1\\ \mathscr{M}_1\end{pmatrix} = \int dx_1 p_1(x_1, t)\begin{pmatrix}1\\ x\end{pmatrix}\oint_{\Sigma_1^B} N_Z\, ds = \int_{a_1^-}^{a_1^+} dx_1 p_1(x_1, t)\, B_1(x_1)\begin{pmatrix}1\\ x\end{pmatrix} = O(\epsilon^2), \quad (47)$$

where the contour integral in the cross-flow plane is carried out only for $z < 0$. The expression for $p_1$ can be simplified by using (15) and by recognizing that the time dependence of the log and arctangent function in (34) occurs via the variable $x_2$. The result is

$$\begin{aligned} p_1(x_1, t)/\rho = {} & \frac{-U_1^2}{4\pi h}\int_{a_1^-}^{a_1^+}\frac{S'(\xi_1)}{x_1-\xi_1}\,d\xi_1 - \frac{U_2^2}{4\pi h}\int_{a_2^-}^{a_2^+}\frac{S'(\xi_2)\,(x_2-\xi_2)}{(x_2-\xi_2)^2+s_p^2}\,d\xi_2 \\ & + \int_{a_2^-}^{a_2^+}\left[\left(\frac{\partial\gamma_2}{\partial t}\right)\tan^{-1}\left(\frac{-s_p}{x_2-\xi_2}\right) - U_2\gamma_2\frac{s_p}{(x_2-\xi_2)^2+s_p^2}\right]d\xi_2 \\ & + U_2\gamma_2(a_2^-, t)\tan^{-1}\left(\frac{-s_p}{x_2-\xi_2}\right), \end{aligned} \quad (48)$$

where one may recognize that the first term is the usual 'sinkage pressure' that body 1 experiences while moving alone in shallow water (Tuck 1966). The second term corresponds to the axial flow generated by the forward motion of body 2 and the last two terms represent the steady and unsteady effects of the adjacent vortex distribution. If $\delta_2$ is $O(1)$, one obtains the following simple expression for $\mathscr{Z}_1$ and $\mathscr{M}_1$:

$$\begin{pmatrix}\mathscr{Z}_1\\ \mathscr{M}_1\end{pmatrix} = \begin{pmatrix}\mathscr{Z}_0\\ \mathscr{M}_0\end{pmatrix} - \frac{U_2^2}{4\pi h}\int_{a_1^-}^{a_1^+} dx_1\begin{pmatrix}1\\ x_1\end{pmatrix} Bx_1\int_{a_2^-}^{a_2^+} d\xi_2 S'(\xi_2)\frac{x_1-s_t-\xi_2}{(x_1-s_t-\xi_2)^2+s_p^2}, \quad (49)$$

where $-\mathscr{Z}_0$ and $\mathscr{M}_0$ are the single-body sinkage force and trimming moment respectively. The additional term on the right can be used to determine the transient heave and pitch motion of ship 1 caused by the motion of ship 2.

The leading-order lateral force $\mathscr{Y}_1$ comes from $P_2$, defined by (46). In differential form,

$$\frac{d\mathscr{Y}_1}{dx_1}(x_1) = \oint_{\Sigma_1^B} P_2(Y_1, Z_1; x_1, t)\, N_Y\, ds, \quad (50)$$

which can actually be expressed in terms of the cross-sectional area and the added-mass characteristics of the cross-section. This was in fact carried out using momentum analysis by Newman (1975) in connexion with the swimming of a slender fish in a deep fluid. Since the extension of his analysis to the case of finite depth is sufficiently straightforward, it does not warrant another derivation here. In the present notation, the results, after correcting for the double-body effect, can be written as

$$\frac{d\mathscr{Y}_1}{dx_1} = 2hC_1(x_1)\frac{\partial'}{\partial t}V_1^* - 2hU_1\frac{\partial}{\partial x_1}(V_1^* C_1) + \frac{U_1}{2}V_1^* S_1' + O(\epsilon^3), \quad (51)$$

where $\mathscr{Y}_1$ represents the sway force on the 'submerged' portion of the body. Integrating (51) along the length of the body, one gets

$$\mathscr{Y}_1 = 2h\frac{\partial'}{\partial t}\int_{a_1^-}^{a_1^+} V_1^*(x_1, t)\, C_1(x_1)\, dx_1 + \frac{U_1}{2}\int_{a_1^-}^{a_1^+} S_1'(x_1)\, V_1^*(x_1)\, dx_1 + 2hU_1 C_1(a_1^-)\, V_1^*(a_1^-, t). \quad (52)$$

In arriving at (52) it was assumed that body 1 was pointed at the bow, hence $C_1(a_1^+) = 0$. The yaw moment $\mathscr{N}_1$ about $O_1$ can be obtained in an analogous manner by noting that $d\mathscr{N}_1 = x_1 d\mathscr{Y}_1$. The final expression is

$$\mathscr{N}_1 = 2h\frac{\partial'}{\partial t}\int_{a_1^-}^{a_1^+} V_1^* C_1 x_1\,dx_1 + 2hU_1\int_{a_1^-}^{a_1^+} V^*\left(C_1+\frac{x_1 S_1'}{4h}\right)dx_1 + 2hUV_1^*(a_1^-,t)C_1(a_1^-)\,a_1^-. \tag{53}$$

Equations (47), (52) and (53) permit the rapid evaluation of the instantaneous force and moment on the individual body once $V^*$ is known. The expediency lies in the fact that only the *overall* sectional characteristics of the body are needed, not a detailed knowledge of the potentials $\Phi^{(1)}$ and $\Phi^{(2)}$.

## 6. Approximate solutions for weakly coupled cases

The complete solution of (37) and (38) for bodies of arbitrary shape requires substantial numerical effort. Before embarking on such a major task it seems worthwhile to test the practical usefulness of the theory for a few simple cases. Towards this end, numerical solutions have been obtained for a few situations that reflect weak coupling.

First, for case 1 in §4, one notes that the sway force and yaw moment can be written in closed form if $\delta_1$ and $\delta_2$ are both $O(1)$. By (37) and (38), or as expected intuitively,

$$V_1^*(x_1,t) = V_{21}^{(m)}(x_2), \quad V_2^*(x_2,t) = V_{12}^{(m)}(x_1), \tag{54}$$

where $V_{21}^{(m)}$ and $V_{12}^{(m)}$ are the normal velocities induced by the source distribution, i.e. the right-hand side of (37) and (38). Next, one observes from (4) that

$$\frac{\partial'}{\partial t}V_{21}^{(m)}(x_2) = \frac{\partial}{\partial x_2}V_{21}^{(m)}\frac{dx_2}{dt} = (U_1-U_2)\frac{\partial V_{21}^{(m)}}{\partial x_1}. \tag{55}$$

Thus it follows from (52) and (53) that

$$\mathscr{Y}_1/\rho = \frac{U_2-U_1}{2}\int_{a_1^-}^{a_1^+} A_1'(x_1)\,V_{21}^{(m)}(x_1,t)\,dx_1 + \frac{U_2}{2}\int_{a_1^-}^{a_1^+} S_1'\,V_{21}^{(m)}\,dx_1 + U_2A_1(a_1^-)\,V_{21}^{(m)}(a_1^-,t), \tag{56}$$

$$\mathscr{N}_1/\rho = \frac{1}{2}\int_{a_1}^{a_1^+}\{[(U_2-U_1)A_1'+U_2S_1']x_1+4hC_1(x_1)\}\,V_{21}^{(m)}(x_1,t)\,dx_1 + \tfrac{1}{2}U_2A_1(a_1^-)V_{21}(a_1^-,t)\,a_1^-. \tag{57}$$

These are the shallow-water analogues of the equations given by Tuck & Newman (1974, §2). With the exception of the functional representation of $V_{21}^{(m)}$ and the values of the added-mass coefficients, the deep- and shallow-water cases are identical.

A less trivial situation would correspond to case 2 discussed in §4, in which one needs to solve the following Prandtl lifting-line equation to obtain $V_1^*(x,t)$:

$$V_1^*(x_1,t) + \frac{1}{\pi}\int_{-\infty}^{a_1+}\frac{(V_1^*C_1)'}{x_1-\xi_1}\,d\xi_1 = V_{21}^{(m)}(x_1,t). \tag{58}$$

The complexity introduced by the wake can be avoided if the following two cases are considered: (*a*) body 1 stationary in space, which corresponds to the situation of a moored vessel disturbed by a passing one; (*b*) both bodies moving at the same speed, which corresponds to two vessels moving in a refuelling configuration. The integro-differential equation for both cases is

$$V_1^*(x_1,t)+\frac{1}{\pi}\int_{a_1^-}^{a_1^+}\frac{(V_1^*C_1)'}{x_1-\xi_1}\,d\xi_1 = V_{21}^{(m)}(x_1,t),$$

with the understanding that there is actually no dependence on time for (*b*). In non-dimensional form, this can be written as

$$V_1^*(\bar{x}_1,t)+\frac{1}{\pi}\int_{-1}^{1}\frac{(V_1^*\bar{C}_1)'}{\bar{x}_1-\bar{\xi}_1}\,d\bar{\xi}_1 = V_{21}^{(m)}(\bar{x}_1,t), \tag{59}$$

where $\bar{x}_1 = [2x_1-(a_1^+-a_1^-)]/L_1$ and $\bar{C}_1 = 2C_1/L_1$. Applying the Cauchy inversion technique (Muskhelishvili 1958, p. 373), one obtains

$$(V_1^*\bar{C}_1)' = \frac{-1}{\pi(1-\bar{x}_1^2)^{\frac{1}{2}}}\int_{-1}^{1}\frac{(1-\bar{\xi}_1^2)^{\frac{1}{2}}(V_1^*-V_{21})}{\bar{\xi}_1-\bar{x}_1}\,d\bar{\xi}_1-\frac{\bar{\Gamma}_1}{2\pi(1-\bar{x}_1^2)^{\frac{1}{2}}}, \tag{60}$$

where $\bar{\Gamma}_1$ is an unknown constant which is related to the bound circulation on body 1:

$$\bar{\Gamma}_1 = \int_{-1}^{1}\gamma_1(x_1,t)\,dx_1 = V_1^*(1)\bar{C}_1(1)-V_1^*(-1)\bar{C}_1(-1). \tag{61}$$

The second term of (60) represents the homogeneous solution of (59) and must be determined by the Kutta condition.

*Case (a): body 1 stationary*

If the body ends are pointed, (61) implies that $\Gamma_1 = 0$. If the ends are square, it seems plausible to assume that the flow at the ends will be diverted by the blockage, hence $V^*(\pm 1,t) = 0$. Again, $\Gamma_1 = 0$. Thus, in either situation, the homogeneous solution can be discarded. Integrating (60) from $x$ to the 'leading edge' one obtains the following Fredholm integral equation of the second kind:

$$V_1^*(\bar{x}_1,t)\bar{C}_1(x_1) = \frac{1}{\pi}\int_{-1}^{1}K(\bar{x}_1,\xi_1)V_1^*(\bar{\xi}_1)\,d\xi_1-\frac{1}{\pi}\int_{-1}^{1}V_{21}^{(m)}(\bar{\xi}_1,t)K(\bar{x}_1,\xi_1)\,d\bar{\xi}_1, \tag{62}$$

where
$$K(x,\xi) = \int_x^1\frac{dx'}{(\xi-x')}\left(\frac{1-\xi^2}{1-x'^2}\right)^{\frac{1}{2}} = \tfrac{1}{2}\log\left[\frac{1-x\xi-(1-\xi^2)^{\frac{1}{2}}(1-x^2)^{\frac{1}{2}}}{1-x\xi+(1-\xi^2)^{\frac{1}{2}}(1-x^2)^{\frac{1}{2}}}\right]. \tag{63}$$

For a general $V_{21}^{(m)}(\bar{\xi}_1,t)$, (62) has to be solved numerically. Once $V_1^*$ has been determined, (52) and (53) can be used to obtain the sway force and yaw moment.

*Case (b): steady motion of two bodies*

By (33), the appropriate end condition is $(V^*C_1)' = 0$ at the trailing edge $\bar{x}_1 = -1$. From (60), this implies

$$\Gamma_1 = -2\int_{-1}^{1}\left(\frac{1-\bar{\xi}_1}{1+\bar{\xi}_1}\right)^{\frac{1}{2}}[V_1^*(\bar{\xi}_1)-V_{21}(\bar{\xi}_1)]\,d\bar{\xi}_1. \tag{64}$$

Substituting the result into (60) and integrating with respect to $x$, one obtains

$$V_1^*(\bar{x}_1,)\bar{C}_1(x_1) = \frac{1}{\pi}\int_{-1}^{1}\hat{K}(\bar{x}_1,\xi_1)[(V_1^*(\bar{\xi}_1)-V_{21}^{(m)}(\bar{\xi}_1)]\,d\xi_1, \tag{65}$$

with
$$\hat{K}(x,\xi) = \int_x^1 \frac{dx'}{\xi - x'}\left(\frac{1-\xi}{1+\xi}\right)^{\frac{1}{2}}\left(\frac{1+x'}{1-\xi}\right)^{\frac{1}{2}}$$
$$= K(x,\xi) - \left(\frac{1-\xi}{1+\xi}\right)^{\frac{1}{2}}(\tfrac{1}{2}\pi - \sin^{-1}x), \tag{66}$$

where to be consistent with (52), the bow has been assumed to be pointed, i.e. $\bar{C}_1(1) = 0$. It can be seen that (65) differs from (62) only by an additional term in the definition of the kernel function. The lateral force and yaw moment can be easily obtained from (52) and (53) with the time-derivative terms discarded.

## 7. Numerical results and discussion

The solutions of (62) and (65) are obtained using the method of discretization, by which the original equation can be written as a system of linear equations:

$$V_1^*(\bar{x}_i,t)\,\bar{C}_1(\bar{x}_i) - \frac{1}{\pi}\sum_{j=1}^{N} K_{ij}\,V_1^*(\bar{x}_j,t) = -\frac{1}{\pi}\sum_{j=1}^{N} K_{ij}\,V_{21}^{(m)}(\bar{x}_j,t) \quad (i = 1,2,\ldots,N), \tag{67}$$

where the influence coefficients $K_{ij}$ are given by

$$K_{ij} = \int_{\bar{\xi}_j}^{\bar{\xi}_{j+1}} K(\bar{x}_i,\bar{\xi})\,d\xi \quad \text{or} \quad = \int_{\bar{\xi}_j}^{\bar{\xi}_{j+1}} \hat{K}(\bar{x}_i,\bar{\xi})\,d\bar{\xi}, \tag{68}$$

$(\bar{\xi}_1, \bar{\xi}_2, \ldots, \bar{\xi}_N)$ denoting the set of grid points along the $x$ axis and $\bar{x}_i$ being the midpoint between two successive grid points. Most of the integrals of (68) can be evaluated analytically.

Figures 4 (*a*) and (*b*) show the computed lateral force and yaw moment acting on a moored 100K DWT tanker owing to the passage of a 30K DWT tanker. The results are plotted *vs*. the relative position of the vessels, which also represents the time axis. The solid lines are speed-averaged experimental values taken from Remery (1974). Two sets of theoretical results are given. The dashed lines correspond to the simple formulae (56) and (57), which do not account for the effects of blockage. Clearly the theoretical predictions are too high. The dotted lines are obtained by solving (62) and using (52) and (53). The peak force and moment are reduced substantially but unfortunately fall below the speed-averaged experimental values. Both sets of computations use the exact values of $C_1(x)$, which are obtained by using a method discussed in Yeung & Hwang (1977), for the lateral-flow problem. These values are depicted in an inset in figure 4 (*b*).

The force and moment acting on a tug boat moving along the side of a cargo vessel in shallow water are shown in figures 5 (*a*) and (*b*). The experimental results are due to Dand (1975). The theoretical predictions are obtained by solving (65) for the larger ship and determining the subsequent cross-flow incident upon the smaller ship from (38). The bow of the tug boat is assumed to be pointed. It is worthwhile to note that while the general behaviour of the experimental curves is predicted fairly well, the peak force and moment are substantially underestimated, particularly when the tug boat is at the stern region of the cargo vessel. This underestimation does not appear to be due to the neglect of viscosity. It was found that using the computed cross-flow and a drag coefficient of 2·0 gives an increase in the predicted value less than 10 % of the experimental value. One may question, of course, the validity of applying an outer theory in

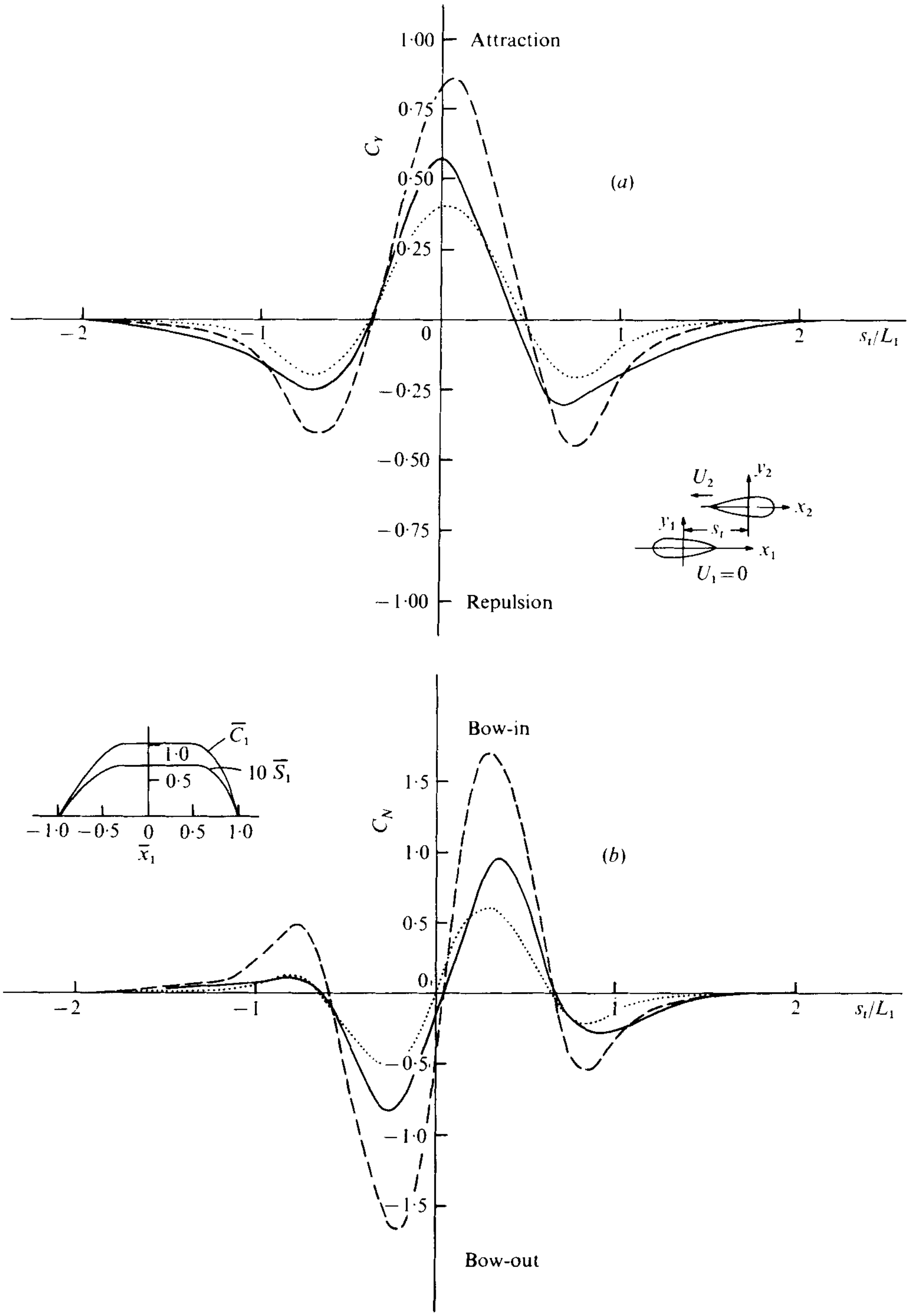


FIGURE 4. (*a*) The sway force and (*b*) the yaw moment acting on a stationary tanker owing to the passage of another. The force coefficient $C_Y$ is defined by $\mathscr{Y}_1/\frac{1}{2}\rho U_2^2 B_1 T_1$ and the moment coefficient $C_N$ by $\mathscr{N}/\frac{1}{2}\rho U_2^2 B_1 T_1$. The geometric parameters are $L_2/L_1 = 0{\cdot}712$, $s_p/L_1 = 0{\cdot}239$, $h/T_1 = 1{\cdot}15$ and $h/T_2 = 1{\cdot}72$. Experimental data are speed-averaged values with $F_h$ ranging from 0·155 to 0·270. ---, theory, $V_1^* = V_{21}^{(m)}$; · · · · ·, theory, $V_1^*$ from (62); ——, experiment, Remery (1974).

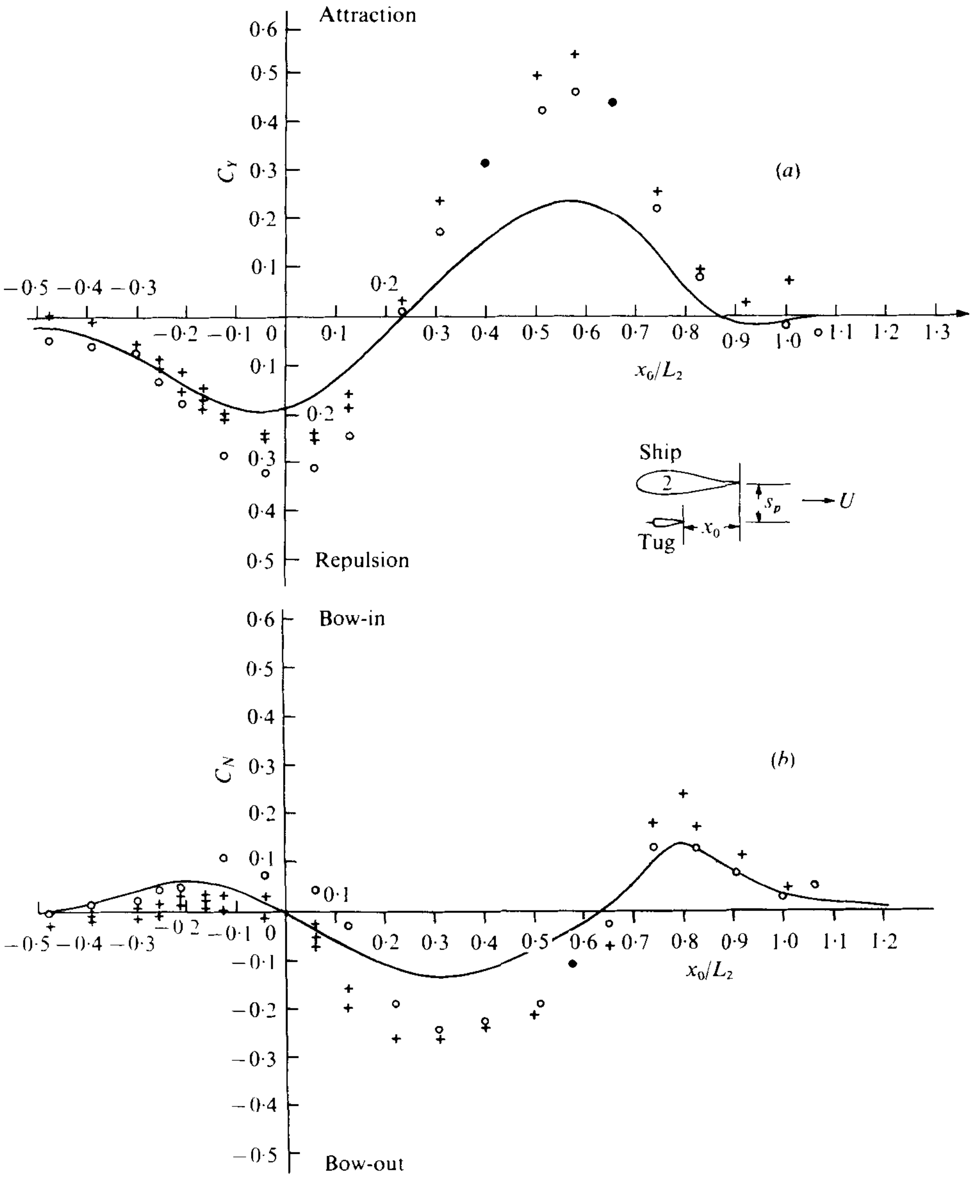


FIGURE 5. (*a*) The sway force and (*b*) the yaw moment acting on a tug moving alongside a cargo ship. Experimental data are for $F_h = 0{\cdot}345$ with $s_p/L_2 = 0{\cdot}128$ and $h/T_2 = 1{\cdot}38$. $C_Y$ is defined by $\mathscr{Y}_1/\frac{1}{2}\rho U_1^2 B_1 T_1$ and $C_N$ by $\mathscr{N}_1/\frac{1}{2}\rho U_1^2 B_1^2 T_1$. ——, theory; +, experiment, tug without screw, Dand (1975); ○, experiment, tug with screw, Dand (1975).

such close proximity. The ratio of the lateral clearance between the two bodies to the length of ship 2 is 0·023. It appears that the main source of error lies in the representation of the body in the outer problem by only a first-order source distribution. These preliminary comparisons, however, show that the theory discussed portrays the *qualitative* features of ship-to-ship interaction quite well even when the separation between ships is not $O(1)$. The case of two ships in steady motion with a separation $O(\epsilon)$ was considered by Yeung & Hwang (1977).

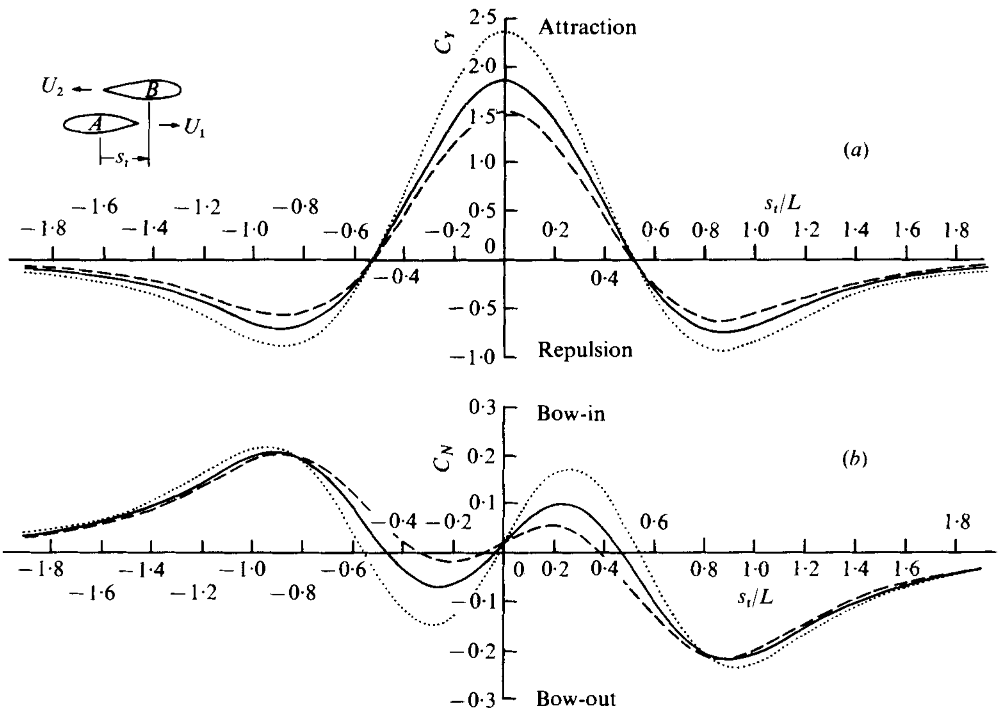


FIGURE 6. (*a*) The sway force and (*b*) the yaw moment acting on two identical vessels in head-on encounter with $h/T = 1{\cdot}11$ and $s_p/L = 0{\cdot}5$. $C_Y$ and $C_N$ are non-dimensionalized by $\frac{1}{2}\rho BT|U_1U_2|$ and $\frac{1}{2}\rho BTL|U_1U_2|$ respectively. ——, $U_2/U_1 = -1{\cdot}0$; ·····, $U_2/U_1 = -1{\cdot}5$, slower ship; – – –, $U_2/U_1 = -1{\cdot}5$, faster ship.

Finally, the theory presented is applied to geometric configurations that are more pertinent to the stated assumptions. Two identical vessels of tanker proportions are assumed to have a parabolic sectional area with rectangular cross-sections. Both vessels are further assumed to have pointed bows and fin-like sterns. The length-to-beam ratio is 6·667 and the beam-to-draught ratio 2·5. The bottom clearance is assumed to be 10 % of the vessel draught. The unsteady interaction force and moment are calculated for a ratio of separation to ship length of 0·5, assuming the flow to be *unblocked* [(56) and (57)].

Figure 6 shows the instantaneous force and moment for two tankers approaching each other. Results for two different speed ratios are given. By moving along the $s_t/L$ axis from right to left, one may visualize these curves as the time history of the interaction force and moment. One observes that during the approach each vessel experiences initially a repulsive force and a bow-out moment. Just as the lateral force becomes attractive the yaw moment becomes bow-in. This corresponds to a highly dangerous situation as far as collision is concerned. After the midships have crossed, the yaw moment changes to bow-out again, but the attractive force remains for some time, which may cause the sterns of the vessels to collide. The several reversals of the sign of the yaw moment are of great concern to the helmsman. Another feature observable from these curves is that the slower ship experiences a larger force and moment than the faster one. This fact is in apparent agreement with what one may observe in the operation of a vehicle on a highway.

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